SYNOPSIS OF THE THESIS

Extending the limits of tractability and intractability for Steiner tree, Domination, and its variants on Structured graphs

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1 Evolution of our research

Much of the research work in the literature identifies polynomial-time solvable (P) or NP-complete (NPC) instances of many combinatorial problems. There is very little research work in the literature that focuses on finding a thin-line separation of P versus NPC instances for the combinatorial problem under consideration. This motivates us to consider the following question:

How much one can reduce the gap between P vs. NPC instances ?

The motivation for our research stems from addressing the above question in the context of the Steiner tree and its friends. To pursue this direction, we considered the following problems.

1. The Steiner tree problem (STREE): Initially we considered the Steiner tree problem on bipartite graphs. It is known that STREE in a chordal bipartite graph is NPC, and finding a minimum Steiner tree on bipartite permutation graphs is polynomial-time solvable. However, the complexity of STREE is not known in the immediate subclass of chordal bipartite graphs, which are popularly known as convex bipartite graphs. Further, convex bipartite graphs are a superclass of bipartite permutation graphs. Thus there is a scope for reducing the gap between P vs. NPC instances of STREE on chordal bipartite graphs and its subclasses. We investigated the complexity of STREE on convex bipartite graphs. Our investigation demonstrates that the complexity of STREE on convex bipartite graphs is polynomial-time solvable, thereby reducing the gap of P vs. NPC instances with respect to chordal bipartite graphs and its subclasses. Convex bipartite graphs can also be seen as 0-star caterpillar convex bipartite graphs. Further, we show that for 1-star caterpillar convex bipartite graphs, STREE is NPC. Thus our exploration identified the thin-line separation for STREE on bipartite graphs with convexity property and this is the best possible dichotomy one can obtain for STREE on subclasses of bipartite graphs.

   Interestingly, our algorithm that solves STREE on convex bipartite graphs also serves as a framework for two other classical problems (i) finding STREE on interval graphs, and (ii) the dominating set problem on convex bipartite graphs.

   This creates a curiosity in us to explore this idea (convex property) on graphs having bipartite-like structures. The natural and immediate choice would be the class of split graphs. It is known that STREE on split graphs is NPC. Our investigation of STREE on split graphs with convexity property on independent set reveals that STREE is NPC on star-convex and comb-convex split graphs, and is polynomial-time solvable on path-convex, triad-convex, and circular-convex split graphs. Thus we reduce the gap between P vs. NPC instances of STREE on split graphs by exploiting convexity properties, in particular, path-convex (unary tree-convex) and comb-convex (binary tree-convex) split graphs. Further, our gap reduction exploration exploits the structure of the convex property and we prove that when the degree of the star in star-convex split graphs is bounded, STREE is polynomial-time solvable, and when the backbone path of the underlying comb-convex bipartite graph is bounded, STREE is polynomial-time solvable. Furthermore, it turns out that STREE on tree-convex split graphs with convexity on K is polynomial-time solvable. Interestingly, for split graphs, a minimum Steiner set is also a minimum dominating (connected, total) set in split graphs. However, the other variants of dominating set, namely the total outer-connected dominating set, and the outer-connected dominating set, which are recently popular in the literature, whose complexity is not known for split graphs with convex properties.

2. The total outer-connected dominating set problem (TOCD): TOCD is NP-complete on split graphs. Further for split graphs, if the clique is bounded, then TOCD is linear-time solvable. We analyze the complexity of TOCD on convex split graphs and prove that: on star-convex split graphs and the comb-convex split graphs, TOCD is NPC, and on path-convex split graphs, TOCD is polynomial-time solvable. The result on comb-convex and path-convex split graphs gives a thin-line separation between P vs. NPC instances of TOCD. Further, we also obtain a thin-line separation of instances of TOCD with respect to the degree of clique vertices in split graphs, that is TOCD is polynomial-time solvable on $\Delta^I = 2$ split graphs, and is NPC on $\Delta^I = 3$ split graphs.

   We extend our study to the outer-connected dominating set problem (OCD), and we prove that OCD is NPC on $\Delta^I = 3$ split graphs.

3. The outer-connected dominating set problem (OCD): OCD is NP-complete on perfect elimination bipartite graphs, whereas finding minimum outer-connected dominating set is polynomial-time solvable on bipartite chain graphs. However, the complexity of OCD in a subclass of perfect elimination bipartite graphs and a superclass of bipartite chain graphs is not known in the literature. We attempt to reduce this gap of OCD in bipartite graphs. Towards this attempt, we prove that OCD is NPC on chordal bipartite graphs and polynomial-time solvable on biconvex bipartite graphs. Although we reduced the gap in bipartite graphs, identifying the thin-line separation for OCD is a challenging task.

   We also explored the computational complexity of TOCD (OCD) in planar graphs which are equally interesting and challenging. Our attempt reveals that TOCD (OCD) on planar graphs is NPC.

4. *Dominating set as functions:* Observe that the dominating set of a graph G can also be visualized as a function h from $V(G)$ to $\{0,1\}$ such that $\forall v \in V(G)$, $\sum_{u \in N_G[v]} h(u) \geq 1$ (for every vertex labeled 0, it must have at least one neighbor which is labeled 1). The objective is to find a dominating function h such that $\sum_{v \in V(G)} h(v)$ is minimum. It is now natural to ask, *can this function be generalized?* Several generalizations of dominating functions have been studied in the literature. We consider a variant of the dominating function, namely the Roman dominating function which is a function from $h : V(G) \rightarrow \{0,1,2\}$.

   (-) The Roman k-domination problem: Roman 1-domination problem which is also known as the Roman domination problem is extensively studied in the literature. The generalization of the Roman 1-domination problem is the Roman k-domination problem (RKDP), whose complexity is not well-studied on structured graphs. We study the complexity status of RKDP on split graphs, bipartite chain graphs, and bipartite graphs having convexity properties such as star, and comb. Further, we present an interesting thin-line separation for RKDF on split graphs which is: RKDP is NP-complete on $K_{1,2k+3}$-free split graphs, and is polynomial-time solvable for RKDF, $k = 1$ on $K_{1,2k+2}$-free split graphs.

5. *Parameterized complexity:* Having done a study on extending the limits of tractability and intractability from the perspective of classical complexity, a natural direction for further research and the state of the art is the study of parameterized complexity theory. We explore the Steiner tree, domination, and its friends from the perspective of parameterized complexity theory. In particular, we establish the following results:

   (i.) The parameterized Steiner tree problem: We prove that this problem is W[2]-hard when parameterized by the solution size, and is fixed-parameter tractable when parameterized by the degree of a vertex in the independent set partition.

   (ii.) The Roman k-domination problem: We show that for split graphs finding RKDF is W[1]-hard with the parameter being the weight of the Roman k-domination function. On the positive side, when $|I|$ is bounded by $|K|$, we show that RKDF is in FPT with the parameter being a clique number. For the Roman 1-domination problem, we prove that fixed-parameter tractability when parameterized by distance to cluster graph.

   (ii.) The Minus domination problem (MDP): We consider another variant of the dominating set function which is the minus dominating function $f : V(G) \rightarrow \{-1,0,1\}$. The classical complexity of the minus domination problem is well-studied in the literature. We explore the parameterized complexity perspective of MDP and prove that MDP is fixed-parameter tractable when parameterized by twin cover, or neighborhood diversity. MDP exhibits a pseudo polynomial-time algorithm when parameterized by cluster vertex deletion set.

6. In the context of the approximation algorithms, we show the following approximable and inapproximable results:

   (i.) We prove that there is no constant factor approximation algorithm for the Roman k-domination problem unless P = NP. Further, we obtain a $\log n$-approximation algorithm for $K_{1,2k+3}$-free split graphs.

(ii.) We prove that the minimum outer-connected dominating set problem for a split graph cannot be approximated within a factor $(1-\epsilon)\ln n$ in polynomial time for any constant $\epsilon > 0$ unless NP $\subseteq$ DTIME($|V|^{O(loglog|V|)}$).

2 Literature at a glance and Research gap identified

1. The Steiner tree problem: The minimum Steiner tree problem (STREE) [1] is a classical subset problem. Given an unweighted connected graph G and $R \subseteq V(G)$, the problem asks for a minimum cardinality set $S \subset V(G)$ such that the set $R \cup S$ induces a connected subgraph. Subsequently using a traversal algorithm such as breadth-first search or depth-first search, one can obtain a tree on $R \cup S$, such a tree is known as the Steiner tree for the terminal set R. The sets R and S are known as the terminal set and the Steiner set, respectively, in the literature.

   On the complexity front, STREE is NP-complete on general and bipartite graphs as there is a polynomial-time reduction from the Exact-3-Cover problem [2]. Further, it is NP-complete on bipartite graphs [3], split graphs [4], and chordal bipartite graphs [5].

   - *Research Gap:* The two well-known subclasses of chordal bipartite graphs are convex bipartite graphs [6], and bipartite distance hereditary graphs [7]. Interestingly, STREE is polynomial-time solvable in bipartite distance hereditary graphs [7], however, to the best of our knowledge, the complexity of STREE in convex bipartite graphs is open.

     Imposing the property convexity on bipartite graphs is a promising direction for further research because many problems that are NP-complete on bipartite graphs become polynomial-time solvable on convex bipartite graphs. Some of the NP-hard reductions restricted to bipartite graphs can be reinforced further by introducing convex properties such as star, comb, tree, etc., For example, the Hamiltonian cycle and Hamiltonian path are NP-hard on star-convex bipartite graphs [8]. While convexity in bipartite graphs seems to be a promising direction in strengthening the existing classical hardness result or in discovering a polynomial-time algorithm, we wish to investigate this line of research for STREE and the dominating set problem (DS) problems restricted to split graphs.

     Further, we consider bisplit graphs which resemble the class of split graphs [9]. As per our knowledge, there are very few combinatorial problems whose complexity status is known on bisplit graphs [10, 11, 12]. We investigate the complexity of STREE on bisplit graphs.

2. The total outer-connected dominating set problem (TOCD): A set D is a dominating set of $V(G)$, if each vertex in $V(G) \setminus D$ is adjacent to at least one vertex in D. A dominating set D is a total dominating set, if each vertex in D is adjacent to at least one vertex in D, and it is an outer-connected dominating set if the graph induced on $V(G) \setminus D$ is connected. A dominating set D is called a total outer-connected dominating set if D is a total dominating set and an outer-connected dominating set. The study on these variants was initiated by Cyman [13],[14] as it has applications in computer networks and facility location problems. On the computational complexity front, domination and its variants are NP-complete in general graphs. In particular, the Total Outer-Connected Domination problem (TOCD) in split graphs is NPC [15].

   - *Research gap:* It appears from the literature that the computational complexity of TOCD on chordal bipartite graphs and its subclasses is open. Further, the only known results on split graphs is the result of [15]. One can also reduce the complexity gap of [15], and consider convex properties with respect to the split graphs.

3. The outer-connected domination problem (OCD): The study of OCD was initiated by Cyman in [13], in which he proved that OCD on bipartite graphs is NP-complete. Further, a minimum outer-connected dominating set can be found in linear time on bipartite chain graphs, bounded tree-width graphs [16], proper interval graphs [17] and polynomial-time solvable on interval graphs [18]. OCD is NP-complete on perfect elimination bipartite graphs, and subclass of chordal graphs such as doubly chordal graphs, undirected path graphs, and split graphs [16, 17, 19].

- *Research gap:* The computational complexity of OCD on chordal bipartite graphs and convex bipartite graphs which is a popular subclass of chordal bipartite graphs is not known in the literature.

4. The Roman k-domination problem: A Roman dominating function on a graph G is a vertex labeling $f : V(G) \to \{0, 1, 2\}$ such that every vertex with label 0 has a neighbor with label 2. The weight $w(f)$ of an RDF f is $\Sigma_{v \in V(G)} f(v)$. The Roman domination number $\gamma_R(G)$ is the minimum weight of an RDF of G. A Roman k-dominating function (RKDF) on G is a function $f : V(G) \to \{0, 1, 2\}$ such that every vertex u for which $f(u) = 0$ is adjacent to at least k vertices $v_1, v_2, \ldots, v_k$ with $f(v_i) = 2$ for $i = 1, 2, \ldots, k$. The weight of a Roman k-dominating function is the value $w(f) = \Sigma_{u \in V(G)} f(u)$. The minimum weight of a Roman k-dominating function on a graph G is called the Roman k-domination number $\gamma_{kR}(G)$ of G.

   On the complexity front, Roman domination is polynomial-time solvable on interval graphs, cographs, and AT-free graphs [20], and Roman domination is NP-complete on bipartite graphs [21, 22]. Finding a minimum weight RKDF is called the minimum weight Roman k-domination problem (RKDP). RKDP is known to be polynomial-time solvable for cactus graphs [23].

   - *Research gap:* The computational complexities of the Roman k-domination problem on split graphs, chordal graphs, chordal bipartite graphs, and bipartite graphs with convex properties are not known in the literature.

5. The minus domination problem (MDP): Given a graph $G = (V, E)$, a *minus dominating function* $f : V(G) \to \{-1, 0, 1\}$ is an assignment of labels to the vertices of G such that for each vertex $v \in V(G)$ the sum of labels assigned to the vertices in the closed neighborhood of v (denoted by $N[v]$) is at least one, i.e., $\sum_{w \in N[v]} f(w) \geq 1$. MDP is NP-complete on chordal bipartite graphs, split graphs, and bipartite planar graphs of degree at most 4 [24, 25, 26, 27]. Given an integer ℓ, finding a minus dominating function of weight ℓ is NP-complete [26]. The problem is polynomial-time solvable on trees, graphs of bounded rank-width, cographs, distance hereditary graphs, and strongly chordal graphs [24, 26].

   - *Research gap:* The parameterized complexity of MDP with respect to structural parameters is not explored in the literature.

3 Motivation

STREE and DS: Are they friends?It is interesting to see that on split graphs, a minimum Steiner set is also a minimum dominating set. Further, it is also a total dominating set as well as a connected dominating set. Thus, at least in the context of split graphs, we see an interesting connection between the Steiner tree and the domination problems. This gave us a spark to explore whether the solution to one can help us in solving the other. We are successful in our attempt to solve the domination problem in split graphs using the Steiner tree problem in split graphs. In this line of thought, we explore further for other variants of domination. While we attempted to solve STREE on convex bipartite graphs, we were able to solve STREE for interval graphs also. This line of thought gave us some more insights into the relationship between STREE and DS and created an urge in us to explore the other variants of domination restricted to split graphs, bipartite graphs, various subclasses, and generalization.

4 Our contributions

We highlight the key results in this section.

1. **Complexity of Steiner tree on bipartite graphs with convex property.**

   A bipartite graph $G(X, Y)$ whose vertex set is partitioned into X and Y is a convex bipartite graph, if there is an ordering of $X = (x_1, \ldots, x_m)$ such that for all $y \in Y$, $N_G(y)$ is consecutive with respect to the ordering of X, and G is said to have convexity with respect to X. A k-star caterpillar is a tree with a collection of stars with each star having k vertices of degree one whose roots are joined by a path. For a bipartite graph with partitions X and Y, we associate a k-star caterpillar on X such that for each vertex in Y, its neighborhood induces a tree.

Theorem 1. *Let G be a convex bipartite graph (0-star caterpillar convex bipartite graph). The minimum Steiner set S of G for the following cases of R is obtained in polynomial time.*

(a) $R = X$
(b) $R = Y$
(c) $R \subseteq X$
(d) $R \subseteq Y$
(e) $R \cap X \neq \emptyset$ *and* $R \cap Y \neq \emptyset$

Theorem 2. *STREE is NP-complete on 1-star caterpillar convex bipartite graphs.*

Thus we obtain the following dichotomy result: STREE is NP-complete for 1-star caterpillar convex bipartite graphs and polynomial-time solvable for 0-star caterpillar convex bipartite graphs. We also strengthen the well-known result of [5], which says STREE in chordal bipartite graphs is NP-complete (reduction instances are 3-star caterpillar convex bipartite graphs). As an application, we use our STREE results to solve: (i) the classical dominating set problem in convex bipartite graphs, and (ii) STREE on interval graphs, linear time.

2. **Complexity of Steiner tree on split graphs with convex property.**

Definition 3. *A split graph G is called* π-convex with convexity on K *if there is an associated structure π on K such that for each $v \in I$, $N_G(v)$ induces a connected subgraph in π.*

Definition 4. *A split graph G is called* π-convex with convexity on I *if there is an associated structure π on I such that for each $v \in K$, $N_G^I(v)$ induces a connected subgraph in π.*

We proved the following theorems by considering the convex property on I.

Theorem 5. *STREE on star-convex and comb-convex split graphs is NPC.*

Theorem 6. *STREE is polynomial-time solvable on path-convex, triad-convex, and circular-convex split graphs.*

It turns out that STREE is polynomial-time solvable on tree-convex split graphs with convexity on K. From the parameterized perspective with solution size being the parameter, we show that the Steiner tree problem on split graphs is $W[2]$-hard, whereas when the parameter is treewidth and the solution size, we show that the problem is fixed-parameter tractable, and if the parameter is the solution size and the maximum degree of I (d), then we show that the Steiner tree problem on split graphs has a kernel of size at most $(2d-1)k^{d-1} + k,\ k = |S|$.

3. **The total outer-connected dominating set problem (TOCD).**
In a split graph, for each vertex u in K, $N_G^I(u) = N_G(u) \cap I$, $d_G^I(u) = |N_G^I(u)|$, and for each vertex v in I, $N_G^K(v) = N_G(v) \cap K$, $d_G^K(v) = |N_G^K(v)|$. For each vertex u in K, $N_G^I[u] = (N_G(u) \cap I) \cup \{u\}$, and for each vertex v in I, $N_G^K[v] = (N_G(v) \cap K) \cup \{v\}$. For a split graph G, $\Delta_G^I = \max\{d_G^I(u)\}, u \in K$ and $\Delta_G^K = \max\{d_G^K(v)\}, v \in I$.

Theorem 7. *TOCD is NPC on star-convex split graphs, and is polynomial-time solvable on comb-convex split graphs.*

Theorem 8. *TOCD is NPC on $\Delta^I = 3$ split graphs, and is polynomial-time solvable on $\Delta^I = 2$ split graphs.*

4. **The outer-connected dominating set problem (OCD).**

Theorem 9. *For planar graphs, OCD is NP-complete.*

Theorem 10. *For chordal bipartite graphs, OCD is NP-complete.*

Theorem 11. *Let G be a biconvex graph. A minimum outer-connected dominating set D of G can be found in polynomial time.*

Theorem 12. *For split graphs, the outer-connected dominating set problem when parameterized with solution size is W[2]-hard.*

Further, from the parameterized complexity front, we show that the parameterized version of outer-connected domination problem with the solution size being the parameter is W[2]-hard for r-regular graphs, and W[1]-hard for bipartite graphs. We show that the parameterized version of outer-connected domination problem is fixed-parameter tractable on r-regular graphs for a fixed r and the solution size.

Furthermore, we prove that the minimum outer-connected dominating set problem for a split graph cannot be approximated within a factor $(1-\epsilon)\ln n$ in polynomial time for any constant $\epsilon > 0$ unless NP $\subseteq$ DTIME($|V|^{O(loglog|V|)}$), and this implies that the problem of finding an approximation algorithm in FPT time is W[2]-hard for the parameterized version of outer-connected domination with the parameter being the solution size. We show that all minimal outer-connected dominating sets can be found in time $\mathcal{O}(1.7159^n)$ time.

5. **Roman k-domination function.**

Theorem 13. *For split graphs, and for any $k \geq 1$, the Roman k-domination problem (RKDP) is NP-complete.*

We investigate the computational complexity of finding a minimum Roman k-dominating function (RKDF) on split graphs. We present an interesting dichotomy for RKDF on split graphs: NP-complete on $K_{1,2k+3}$-free split graphs, and polynomial-time solvable for RKDF, $k = 1$ on $K_{1,2k+2}$-free split graphs.

Theorem 14. *For star-convex bipartite graphs, and for any $k \geq 1$, the Roman k-domination problem is NP-complete.*

Theorem 15. *For comb-convex bipartite graphs, and for any $k \geq 1$, the Roman k-domination problem is NP-complete.*

Further, we also show that finding RKDF on bipartite chain graphs is polynomial-time solvable, which is a non-trivial subclass of comb-convex bipartite graphs. From the approximation perspective, we show that there is no constant factor approximation algorithm for RKDF and we present $\log n$-approximation algorithm for RKDF. On the parameterized front, we show the following.

Theorem 16. *The Roman domination problem when parameterized by the solution size is W[2]-complete on split graphs.*

Corollary 1. *The Roman domination problem when parameterized by the solution size is W[2]-complete on chordal graphs.*

Theorem 17. *The Roman domination problem parameterized by cluster vertex deletion set can be solved in $O^*(3^r)$ time, where r is the distance to the cluster graph (O^* hides the polynomial factor involved).*

6. **The minus domination problem (MDP).**

Definition 18 (Twin-cover [28])**.** *Given a graph G, a set $S \subseteq V(G)$ is called as* twin cover *of G if the following conditions hold: (i) $G[V \setminus S]$ is a disjoint union of cliques, and (ii) each pair of vertices of a clique in $G[V \setminus S]$ are true twins in G. We then say that G has twin cover number k if k is the minimum possible size of a twin cover of G.*

Theorem 19. *MDP can be solved in $2^{\mathcal{O}(k \cdot 2^k)} n^{\mathcal{O}(1)}$ time, where k is the twin cover number of the graph.*

Definition 20 (Distance to cluster)**.** *A* cluster graph *is a disjoint union of cliques. Given a graph G, a set of vertices $S \subseteq V(G)$ is called a* cluster vertex deletion set *of G if $G - S$ is a cluster graph. The size of the smallest set S for which $G - S$ is a cluster graph is referred to as* distance to cluster.

Theorem 21. *MDP can be solved in $g(k) \cdot n^{2k+6}$, where k is the distance to the cluster number.*

Definition 22 (Neighborhood diversity [29])**.** *Let $G = (V, E)$ be a graph. Two vertices $u, v \in V(G)$ are said to have the* same type *if and only if $N(u) \setminus \{v\} = N(v) \setminus \{u\}$. A graph G has neighborhood diversity at most t, if there exists a partition of $V(G)$ into at most t sets $V_1, V_2, \ldots, V_t$ such that all the vertices in each set have the same type.*

Theorem 23. *MDP can be solved in $t^{\mathcal{O}(t)} n^{\mathcal{O}(1)}$ time, where t is the neighborhood diversity of the graph.*

5 Organization of the thesis

The central theme of this thesis is to study algorithms mainly for the Steiner tree problem (STREE), the dominating set problem (DS), and its variants. Chapter 1 provides a brief overview of key concepts and definitions that are necessary for the chapters that follow. Further, the chapter gives a motivation for problems considered in this thesis, and presents the necessary preliminaries that are required for this thesis.

In Chapter 2, we present our initial research on the Steiner tree problem (STREE) on convex bipartite graphs. This chapter describes the dichotomy with respect to convexity, which shows that STREE on 0-star caterpillar convex bipartite graphs is polynomial-time solvable, whereas on 1-star caterpillar convex bipartite graphs the problem is NP-complete.

Chapter 3 serves as an extension to Chapter 2, which is about STREE on convex split graphs. The chapter provides a detailed result that is: STREE is NPC on star-convex and comb-convex split graphs, and is polynomial-time solvable on path-convex, triad-convex, and circular-convex split graphs.

In Chapter 4, we show that the complexity of STREE on bipslit graphs is NPC, and we also analyze the complexity of the dominating set and its variants on bisplit graphs. Further, from the realm of parameterized complexity, we prove that STREE is W[2]-hard on bisplit graphs when parameterized with the solution size, and is fixed-parameter tractable when the parameter is the size of the biclique. Furthermore, we show that the bisplit vertex deletion problem is fixed-parameter tractable.

In Chapter 5, we analyze the complexity of the total outer-connected domination problem on convex split graphs, and the outer-connected dominating set problem on a subclass of bipartite graphs.

The Roman k-domination problem on split graphs and in subclasses of bipartite graphs is presented in Chapter 6.

In Chapter 7, we present our results on the minus dominating function from the parameterized complexity perspective.

In Chapter 8, we summarize the results obtained in this thesis. Finally, we also highlight possible directions for the problems considered in this thesis.

6 List of publications

Conference Proceedings

- Mohanapriya, A., P. Renjith, and N. Sadagopan. "Roman k-Domination: Hardness, Approximation and Parameterized Results." In International Conference and Workshops on Algorithms and Computation (WALCOM), pp. 343-355. Cham: Springer Nature Switzerland, 2023.
- Mohanapriya, A., P. Renjith, and N. Sadagopan. "Short Cycles Dictate Dichotomy Status of the Steiner Tree Problem on Bisplit Graphs." In Conference on Algorithms and Discrete Applied Mathematics (CALDAM), pp. 219-230. Cham: Springer International Publishing, 2023.
- Mohanapriya, A., P. Renjith, and N. Sadagopan. "P Versus NPC: Minimum Steiner Trees in Convex Split Graphs." In Conference on Algorithms and Discrete Applied Mathematics (CALDAM), pp. 115-126. Cham: Springer International Publishing, 2022.

Conference presentations

- A.Mohanapriya, P.Renjith, N.Sadagopan. "Total Outer-Connected Domination in Split graphs - A dichotomy", 36th Annual Ramanujan Mathematical Society Conference (RMS 2021)
- A.Mohanapriya, P.Renjith, N.Sadagopan. "Total Outer-Connected Domination on convex split graphs - Complexity Results", 36th Annual Ramanujan Mathematical Society Conference (RMS 2021)
- A.Mohanapriya, P.Renjith, N.Sadagopan. "Roman domination and its variants in split graphs" In online International Workshop on Domination in Graphs (IWDG 2021).

Journals

- Aneesh, D. H., A. Mohanapriya, P. Renjith, and N. Sadagopan. "Steiner tree in k-star caterpillar convex bipartite graphs: a dichotomy." Journal of Combinatorial Optimization 44, no. 2 (2022): 1221-1247. (Web of Science (SCI))
- Mohanapriya, A., P. Renjith, and N. Sadagopan. "Domination and its variants in split graphs-P versus NPC dichotomy." The Journal of Analysis 31, no. 1 (2023): 353-364. (Web of Science (ESCI))

Papers under review

- A. Mohanapriya, P. Renjith, and N. Sadagopan. "On Convexity in Split graphs: Complexity of Steiner tree and Domination."
- A. Mohanapriya, P. Renjith, and N. Sadagopan. "Roman k-domination: Split graphs and Bipartite graphs."
- A. Mohanapriya, P. Renjith, and N. Sadagopan."Total Outer-Connected Domination on convex split graphs - Classical, Parameterized and Approximation Complexity Results."

References

[1] Michael R Garey. *Computers and intractability: A Guide to the Theory of NP-Completeness.* WH Freeman & Co, 1979.

[2] Frank K Hwang, Dana S Richards, Pawel Winter, and P Widmayer. The Steiner Tree Problem, Annals of Discrete Mathematics, Volume 53. *ZOR-Methods and Models of Operations Research*, 41(3):382, 1995.

[3] S Louis Hakimi. Steiner's problem in graphs and its implications. *Networks*, 1(2):113–133, 1971.

[4] Kevin White, Martin Farber, and William Pulleyblank. Steiner trees, connected domination and strongly chordal graphs. *Networks*, 15(1):109–124, 1985.

[5] Haiko Müller and Andreas Brandstädt. The NP-completeness of Steiner tree and dominating set for chordal bipartite graphs. *Theoretical Computer Science*, 53(2-3):257–265, 1987.

[6] Peter Damaschke, Haiko Müller, and Dieter Kratsch. Domination in convex and chordal bipartite graphs. *Information Processing Letters*, 36(5):231–236, 1990.

[7] Alessandro D'Atri and Marina Moscarini. Distance-hereditary graphs, Steiner trees, and connected domination. *SIAM Journal on Computing*, 17(3):521–538, 1988.

[8] Hao Chen, Zihan Lei, Tian Liu, Ziyang Tang, Chaoyi Wang, and Ke Xu. Complexity of domination, hamiltonicity and treewidth for tree convex bipartite graphs. *Journal of Combinatorial Optimization*, 32(1):95–110, 2016.

[9] Andreas Brandstädt, Peter L Hammer, Vadim V Lozin, et al. Bisplit graphs. *Discrete Mathematics*, 299(1-3):11–32, 2005.

[10] Chakradhar Padamutham and Venkata Subba Reddy Palagiri. Complexity of Roman 2-domination and the double Roman domination in graphs. *AKCE International Journal of Graphs and Combinatorics*, 17(3):1081–1086, 2020.

[11] Litao Guo and Bernard LS Lin. Vulnerability of super connected split graphs and bisplit graphs. *Discrete & Continuous Dynamical Systems-S*, 12(4&5):1179, 2019.

[12] Matthieu Rosenfeld, Giacomo Kahn, and Laurent Beaudou. Bisplit graphs satisfy the Chen-Chvátal conjecture. *Discrete Mathematics & Theoretical Computer Science*, 21, 2019.

[13] Joanna Cyman. The outer-connected domination number of a graph. *Australasian journal of Combinatorics*, 38:35–46, 2007.

[14] Joanna Cyman. Total outer-connected domination in trees. *Discussiones Mathematicae Graph Theory*, 30:377–383, 2010.

[15] BS Panda and Arti Pandey. Complexity of total outer-connected domination problem in graphs. *Discrete Applied Mathematics*, 199:110–122, 2016.

[16] Bhawani Sankar Panda and Arti Pandey. Algorithm and hardness results for outer-connected dominating set in graphs. In *International Workshop on Algorithms and Computation*, pages 151–162. Springer, 2014.

[17] J. Mark Keil and D. Pradhan. Computing a minimum outer-connected dominating set for the class of chordal graphs. *Information Processing Letters*, 113(14):552–561, 2013.

[18] Chiou-Jiun Lin, Jia-Jie Liu, and Yue-Li Wang. Finding outer-connected dominating sets in interval graphs. *Information Processing Letters*, 115(12):917–922, 2015.

[19] Dinabandhu Pradhan. On the complexity of the minimum outer-connected dominating set problem in graphs. *Journal of Combinatorial Optimization*, 31(1):1–12, 2016.

[20] Mathieu Liedloff, Ton Kloks, Jiping Liu, and Sheng-Lung Peng. Roman domination over some graph classes. In *International Workshop on Graph-Theoretic Concepts in Computer Science*, pages 103–114. Springer, 2005.

[21] Chun-Hung Liu and Gerard J Chang. Roman domination on strongly chordal graphs. *Journal of Combinatorial Optimization*, 26(3):608–619, 2013.

[22] Chakradhar Padamutham and Venkata Subba Reddy Palagiri. Algorithmic aspects of roman domination in graphs. *Journal of Applied Mathematics and Computing*, 64(1):89–102, 2020.

[23] Karsten Kammerling and Lutz Volkmann. Roman k-domination in graphs. *Journal of the Korean Mathematical Society*, 46(6):1309–1318, 2009.

[24] Jean Dunbar, Wayne Goddard, Stephen Hedetniemi, Alice McRae, and Michael A. Henning. The algorithmic complexity of minus domination in graphs. *Discrete Applied Mathematics*, 68(1):73–84, 1996.

[25] Peter Damaschke. Minus domination in small-degree graphs. *Discrete Applied Mathematics*, 108(1):53–64, 2001. Workshop on Graph Theoretic Concepts in Computer Science.

[26] Luérbio Faria, Wing-Kai Hon, Ton Kloks, Hsiang-Hsuan Liu, Tao-Ming Wang, and Yue-Li Wang. On complexities of minus domination. *Discrete Optimization*, 22:6–19, 2016. SI: ISCO 2014.

[27] Chuan-Min Lee and Maw-Shang Chang. Variations of Y-dominating functions on graphs. *Discrete Mathematics*, 308(18):4185–4204, 2008.

[28] R. Ganian. Twin-cover: Beyond vertex cover in parameterized algorithmics. *In: Marx, D., Rossmanith, P. (eds) Parameterized and Exact Computation. IPEC 2011. Lecture Notes in Computer Science*, 7112, 2012.

[29] Michael Lampis. Algorithmic meta-theorems for restrictions of treewidth. *Algorithmica*, 64(1):19–37, sep 2012.