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Synopsis Of

Stability Analysis of Externally Interfered Nonlinear Discrete-Time Systems Via Passivity Approach

A Thesis

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1 Abstract

Digital signal processing (DSP) has profoundly impacted diverse scientific and engineering disciplines in recent decades. It involves the application of mathematical algorithms and techniques to manipulate signals acquired from the real world. The advancements made in DSP have brought about revolutionary changes in multiple fields such as space technology, medical imaging, commercial applications, communications systems, radar technology, industrial processes, and scientific research [3, 22, 26]. One of the key components of DSP is digital filters, which have played a crucial role in making this technology so prominent today. DF are used to process and enhance signals by removing unwanted noise and distortions. They are capable of achieving extraordinary performance by efficiently isolating and extracting the desired information from the signals.

Analyzing stability in nonlinear digital systems becomes crucial when considering the limitations imposed by hardware overflow constraints. In the hardware implementation of the digital filter, allocating a finite amount of precision to each number is necessary. This means that arithmetic computations must be modified after a certain number of digits. When performing arithmetic operations such as additions and multiplications in digital filters, the wordlength required for the operation often increases [8, 9, 17]. A wordlength reduction strategy is necessary to avoid the signals from accumulating an ever-increasing wordlength. This strategy typically involves quantization and overflow correction. Quantization affects only the least significant bits of a number, while overflow correction modifies the most significant and least significant bits. It is important to note that overflow nonlinearities, which occur due to exceeding the maximum representable value, can cause severe distortion compared to quantization nonlinearities.

The practical implementation of digital filters is often exposed to unfavorable environments such as external interference [20, 27]. In many cases, higher-order discrete-time systems are implemented by combining multiple lower-order discrete-time systems. However, this cascading process generates interference among the lower-order systems, leading to disastrous effects on the overall performance and stability of the system. This interference can cause a deprivation in performance and even result in system instability.

In a different research context, the phenomenon of "ringing" presents another challenge, leading to undesired effects from past excitations and subsequent oscillations [2, 4, 10, 11, 14, 12]. This quantitative method provides a measure of the performance of a discrete-time system with reference to external input. Failing to examine and mitigate ringing in the system design can result in poor performance or malfunction of the system. Hence, it is crucial to establish suitable criteria for designing digital filters in the presence of nonlinearities and external disturbances.

2 Objectives

This thesis delves into the intricate aspects of finite wordlength nonlinearities and external interference, presenting a comprehensive set of criteria for the stability and performance analysis of discrete-time systems implemented on fixed-point digital hardware. It explores various techniques to address overflow nonlinearities, including saturation, zeroing, triangular, and two's complement. Further, correction methods for quantization nonlinearity, such as roundoff, value truncation, and magnitude truncation, are thoroughly investigated.

The primary objectives of this thesis are outlined as follows:

(a) To propose stability criteria to assess the asymptotic stability of externally interfered discrete-time systems by employing a very strict passivity approach in the presence of saturation nonlinearity, generalized overflow nonlinearities, and under various concatenations of quantization and overflow nonlinearities.

(b) To analyze the stability behavior of externally interfered discrete-time systems using a strict passivity approach in the presence of saturation nonlinearity. The proposed approach is extended to study of passive behavior of discrete-time systems under the effects of finite register length nonlinearities and generalized overflow nonlinearities.

(c) To obtain stability criteria that can eliminate the memory effects of digital filters in the presence of saturation nonlinearity and external disturbances through Hankel norm approach. Further, a stability criteria is derived to analyze the asymptotic behavior of external interfered digital filters in the presence of generalized overflow nonlinearities and quantization/ overflow nonlinearities.

(d) To propose a new criterion for analyzing the stability of Lipschitz nonlinear discrete-time systems with saturation nonlinearity and external interference. The proposed approach is extended to analyze the passive behavior of Lipschitz nonlinear discrete-time systems with generalized overflow nonlinearities and under various concatenations of finite register length nonlinearities.

3 Existing Gaps Which Were Bridged

Passivity is a fundamental characteristic of a dynamical system that is closely related to the energy consumption of the system. It has been widely used in various fields to analyze the stability of different types of systems, such as hybrid systems, neural networks, and Markov jump systems. The passivity approach is an effective technique to examine the stability of discrete dynamical systems implemented on a finite wordlength machine. While a limited number of works in the literature have explored passivity-based conditions for discrete-time systems with finite wordlength nonlinearities [5, 6, 16, 21], there remains room for improvement in passivity results. In this thesis, we propose a very strict passivity criterion for discrete-time systems with external disturbances and saturation nonlinearities. The introduced linear matrix inequality (LMI)-based condition is demonstrated to be more relaxed than existing results [5, 7, 21]. Furthermore, we observe a gap in previous research concerning the presence of quantization and overflow nonlinearities, which significantly impact discrete-time systems in practical scenarios. Hence, we present a stability criterion to achieve the asymptotic stability of externally interfered discrete-time systems by employing a very strict passivity approach.

Moreover, a gap in the literature is identified in a thorough investigation of exponential stability analysis for discrete-time systems [15, 18, 19]. Consequently, we introduce stability criteria that ensure the exponential stability of externally interfered discrete-time systems through a very strict passivity approach and overflow nonlinearities. Further, various criteria are developed to address generalized overflow nonlinearities. Additionally, we also extend our approach to investigate externally interfered discrete-time systems under the combined effects of quantization and overflow nonlinearities.

Furthermore, the criteria discussed in [2, 4, 14] focuses on evaluating the effectiveness of discrete-time systems with saturation nonlinearity and external input using the Hankel norm. However, there is still scope for improvement in this field. We have introduced stability criteria to assess robust performance using the 'Hankel norm' approach to address this gap. By employing sector nonlinearity and Lyapunov functions, along with a very strict passivity approach, sufficient conditions are proposed for the stability investigation of externally interfered nonlinear discrete-time systems.

Due to the importance of nonlinear systems in practical applications, we have also obtained a stability criterion for analyzing the stability of the Lipschitz nonlinear discrete-time systems with finite register length nonlinearities and external interference [1, 13, 14, 23, 25, 28]. The proposed approach for stability investigation of Lipschitz nonlinear discrete-time systems with external disturbance utilizes sector nonlinearity, quadratic Lyapunov functions, and Lipschitz condition. By employing these techniques, sufficient conditions are derived to ensure asymptotic stability through a very strict passivity approach.

4 Most Important Contributions

The following are the significant contributions of this thesis:

(a) A very strict passivity criterion is established for discrete-time system with external disturbance and saturation nonlinearities by employing Lyapunov energy function and sector based characteristics of saturation arithmetic. The proposed LMI-based condition is shown to be more relaxed than existing results [5, 7]. Subsequently, diverse stability criteria are derived to assess asymptotic behavior in the presence of various nonlinearities, including saturation, zeroing, triangular, and two's complement, using a very strict passivity approach. Additional conditions are developed to analyze stability under the effects of quantization and overflow nonlinearities. All the proposed conditions guarantee the asymptotic stability of discrete-time systems under zero external disturbance. The reported results offer a larger stability region and less conservatism when compared to existing literature.

(b) A method to assess the exponential stability and passive behavior of externally interfered discrete-time systems with saturation overflow arithmetic is formulated using quadratic Lyapunov function and sector properties of saturation nonlinearity. Sufficient conditions are established for the exponential stability of an interfered discrete-time system subjected to external disturbance and saturation nonlinearities. The proposed method also introduces diverse stability conditions that take into account various types of generalized overflow nonlinearities, including saturation, zeroing, triangular, and two's complement, as well as scenarios where both quantization and overflow nonlinearities are present.

(c) A novel approach to reduce the undesired memory effects of digital filters associated with saturation based on the Hankel norm performance via a very strict passivity approach is introduced. A technical error in [24] is pointed out. A corrected and improved version over [24] is presented by constructing an appropriate Lyapunov function and better structural properties of saturation arithmetic. An unified criteria is put forth for the stability analysis and performance of externally interfered digital filters with generalized overflow nonlinearity and finite register length nonlinearities, by utilizing a very strict passivity approach. The proposed criteria ensure asymptotic behavior under zero disturbance and passive behavior in the presence of external disturbance.

(d) A new criterion to analyze the asymptotic behavior of a Lipschitz nonlinear discrete-time system is proposed. This criterion takes into account the presence of saturation nonlinearity, external disturbance, and inherent nonlinear components for asymptotic stability investigation of the system through a very strict passivity approach. The approach is extended to analyze the passive behavior of Lipschitz nonlinear discrete-time systems with generalized overflow nonlinearities and under various concatenations of finite register length nonlinearities, providing a comprehensive analysis of these complex systems.

5 Conclusions

In this thesis, the stability of discrete-time systems with finite wordlength nonlinearities and external disturbance has been investigated using a passivity-based approach. The following are the key highlights of the proposed work.

(a) An asymptotic stability condition based on a very strict passivity approach has been established for the externally interfered discrete-time systems through saturation nonlinearity. Further, the proposed method has been extended to derive various stability conditions based on generalized overflow nonlinearities as well as under various concatenations of quantization and overflow nonlinearities. Theoretical validations are supported by appropriate numerical examples, including practical applications.

(b) An exponential stability condition has been derived for the discrete-time systems in the absence of disturbance and passive behavior in the presence of disturbance when influenced by saturation arithmetic and external disturbance. A novel criteria has also been developed to mitigate the effects of limit cycle oscillation in externally interfered discrete-time systems in the presence of finite wordlength nonlinearities and generalized overflow nonlinearities. The utility of the presented approach has been highlighted with illustrative examples.

(c) A technical error in [24] has been identified. A corrected and improved version of [24] has been proposed for a discrete-time system in the presence of external disturbance and saturation overflow nonlinearity. Further, the proposed approach has been explored to obtain various stability criteria that satisfy generalized overflow nonlinearities and quantization/ overflow nonlinearities for interfered discrete-time systems. Numerical simulations have been given to establish the superiority of the proposed results.

(d) A new stability condition has been derived for Lipschitz nonlinear discrete-time systems to analyze the passive behavior in the presence of disturbance and saturation nonlinearity. Several conditions have been established to analyze the asymptotic stability of Lipschitz nonlinear systems for generalized overflow nonlinearities and when both quantization and overflow nonlinearities exists. The utility of the presented approach has been highlighted with illustrative examples.

6 Organization of the Thesis

The proposed outline of the thesis is as follows:

(a) Chapter 1: Introduction and Literature Survey

(b) Chapter 2: Asymptotic Stability Analysis of Discrete-time Systems Using a Very Strict Passivity Approach

(c) Chapter 3: Exponential Stability Analysis of Discrete-time Systems Using a Strict Passivity Approach

(d) Chapter 4: Stability Analysis of Externally Interfered Digital Filters Using Hankel Norm Performance

(e) Chapter 5: Stability Analysis of Lipschitz Nonlinear Discrete-time Systems Via a Very Strict Passivity Approach

(f) Chapter 6: Conclusions and Future Directions

7 List of Publications

List of DOIs of any published or presented work

- **Journal Publications based on Thesis**
  (a) M. Pulikonda and P. Kokil, "Investigation of Memory Effects in Externally Interfered Nonlinear Digital Filter via Passivity Approach," *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2024. (Accepted for Publication) [SCI, IF: 4.4]

  (b) M. Pulikonda and P. Kokil, "Strict Passivity Criterion for Lipschitz Nonlinear Discrete-time System with Saturation Overflow Arithmetic," *Circuits, Systems, and Signal Processing*, 2024. (Accepted for Publication) [SCI, IF: 2.3]

  (c) M. Pulikonda and P. Kokil, "Stability of Interfered Discrete-Time System with Concatenations of Quantization and Overflow," *Circuits, Systems, and Signal Processing*, Vol. 43, pp. 1-16, 2023. doi:10.1007/s00034-023-02467-3. [SCI, IF: 2.3]

  (d) M. Pulikonda, C. G. Parthipan and P. Kokil, "A Linear Matrix Inequality-based Criterion for Realizing Externally Interfered Discrete System using Saturation Arithmetic," *Transactions of the Institute of Measurement and Control*, Vol. 45, no. 5, pp. 921-930, 2023. doi:10.1177/01423312221122468. [SCI, IF: 1.8]

- **Communicated Journal Publications based on Thesis**
  (a) M. Pulikonda and P. Kokil, "Stability of Discrete-Time System Employing Quantization and Overflow Nonlinearities," *Transactions of the Institute of Measurement and Control*. (Submitted after Minor Revision) [SCI, IF: 1.8]
  (b) M. Pulikonda and P. Kokil, "Passivity Based Approach for Exponential Stability of Discrete Time System Under Overflow Nonlinearities," *Journal of Control, Automation and Electrical Systems*. (Submitted after Major Revision) [ESCI, IF: 1.5]
  (c) M. Pulikonda and P. Kokil, "Comments on 'Hankel Norm Based Strict Passivity Performance of Digital Filter Using Saturation Nonlinearities'," *Wireless Personal Communications.* (Under Review) [SCI, IF: 2.2]

- **Conference Presentation based on Thesis**
  (a) M. Pulikonda and P. Kokil, "An Enhanced Stability Criterion for Strict Passive Analysis of Digital Filters," *2020 IEEE 17th India Council International Conference (INDICON)*, Delhi, India, 2020, pp. 1-5, (2020).

- **Other Conference Presentations**
  (a) M. Pulikonda and P. Kokil, "New Results on Generalized Dissipativity Analysis of Discrete System with Overflow Nonlinearity," *Eighth IFAC International Conference on Advances In Control and Optimization of Dynamical Systems (ACODS 2024)*, Delhi-NCR. (Presented)
  (b) S. Jogi, M. Pulikonda and P. Kokil, "Stability and Performance Analysis of Interfered Digital Systems with Generalized Overflow Nonlinearities," *Eighth IFAC International Conference on Advances In Control and Optimization of Dynamical Systems (ACODS 2024)*, Delhi-NCR. (Presented)
  (c) M. Pulikonda and P. Kokil, "Exponential Stability Criterion for State-space Interfered Discrete System through External Disturbance," *2023* 5^{th} *International Conference on Power, Control & Embedded Systems (ICPCES)*, Allahabad, India, 2023, pp. 1-6, (2023).

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